Introduction to electrostatics

October 18, 2025

1 Background

Time has come for us to move to a brand new area of physics, *electromagnetism.* That is, the study of both electricity and magnetism. As is custom, we will start with electricity, as it is simpler in many ways.

The fundamental laws of electromagnetism are given by *Maxwell's equations.* However, it is often convenient to start with a slightly different paradigm (which is also how things were developed chronologically) which puts *Coulomb's force law* in the centre[1].

Coulomb's law states that the force between two electric charges q_1, q_2 a distance r apart is given by:

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

where $\epsilon_0 = 8.85 \times 10^{-11}$ F/m. You may have seen the constants in Coulomb's law wcombined into one k, where $k = \frac{1}{4\pi\epsilon_0}$, which is the same statement. This is where we will start our study of electricity. Some observations about Coulomb's law:

1. It is functionally equivalent to Newton's Universal Law of Graviation. Hence, we should expect a lot of expressions and equations of motion in electrodynamics to be similar to those in graviational fields.

2. Different from gravity, in Coulomb's law the *charge* takes the place of being the central unit of electricity, whereas for gravity it was *mass.* (Notice how q_1, q_2 in Coulomb's law replace m_1, m_2 in Newton's Law of Gravitation).

3. Different from gravity, in Coulomb's law, charges can be both positive and negative (mass can only be positive). Hence, electricity is more flexible, and forces can both repel and attract.

4. Force is actually a vector, so the correct way of writing Coulomb's law would be:

   $$\mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$$

   where is the unit vector from one charge to the other. This will be important to keep in mind in cases where you are summing together forces which are pointing in different directions!

   In this handout, we will start by applying Coulomb's law to solve a bunch of questions in electrostatics (and dynamics). This is the first time where you will really get to put the fundamental things you learned in mechanics to the test in a completely new arena. There is now a different fundamental law governing yourr systems (Coulomb's), but everything that we have learned in *Mechanics*, is still true, and will be useful to solve these problems: conservation of energy, conservation of momentum, generalized coordinate, et cetera.

[1] Coulomb's force law is an equivalent statement to Gauss' law, which is the first of Maxwell's equations. We will use Gauss' law plenty in a subsequent handout!

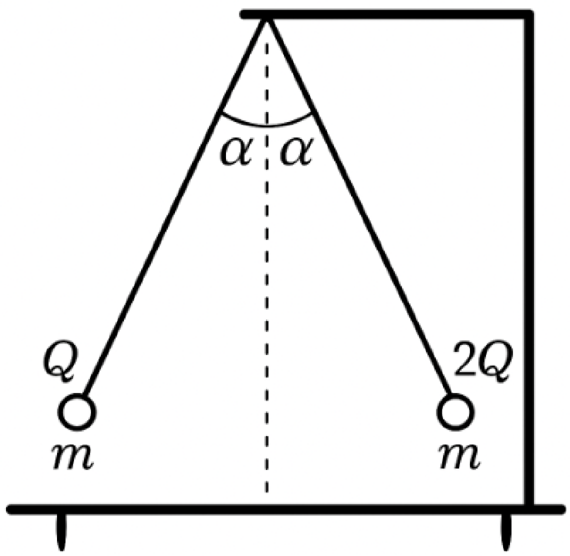

Figure 1: Charged balls

2 Questions

1. Two charges of 1 Coulomb are placed 1 meter apart.
   (a) What is the magnitude of the force between the charges?
   (b) To generate an equally large force due to gravity, what should the mass of the bojects be (assuming they stay 1 meter from each other)?

2. This question is about deriving the expression for electrostatic potential.
   (a) Write down the expression for the Coulomb force between two charges q_1 and q_2, with distance r between them.
   (b) Imagine moving the charge q_1 a small distance dr. Write down an expression for the work dW.
   (c) The total work done moving the charge q_1 from infinity far away to the distance r from charge q_2 can be written as
   $$W = \int_{\infty}^{r} dW$$
   Express the integral in terms of r, dr, q_1, q_2 and ϵ_0
   (d) Calculate the integral to find W.
   (e) We define the potential as the amount of work done per unit charge in moving charge q_1 from infinity to a distance r. Calculate the potential at the point where q_1 is located.

3. Two small metal balls of mass $m = 0.1$ g are suspended from the same point by insulated threads of length $l = 30$ cm (see figure 1. One of the balls has twice the charge of the other. We hold the balls as shown so that both threads make an angle $\alpha = 20°$ with the vertical (both threads lie in the same plane). The balls are released simultaneously, and the angle between the two threads reaches a maximum value of $\beta = 84°$.

   What is the charge of the two balls?

4. Two charged particles, one with mass M and charge Q, and the other with charge $-q$, are placed in a uniform electric field E. After they are released, they remain at a constant distance from each other. What is this distance?

5. Two protons and two positrons ("electrons" with positive charge) lie at the corners of a square with side r. The protons (and therefore also the positrons) lie diagonally opposite each other. Then the particles are released. What is the ratio between the kinetic energy of a positron and a proton when they have come very far apart? An approximate expression is sufficient, and you may assume that the mass of the proton is much larger than the mass of the positrons.

6. A neutron at rest decays into a proton and an electron. The energy released gives the proton and electron kinetic energy. The mass of the proton is 1836 times the mass of the electron. What proportion of the energy released goes into the kinetic energy of the proton?

7. A charged particle with charge q and mass m is given a kinetic energy K_0 when it is in the center of a uniformly charged spherical region with total charge Q and radius R. Q and q have opposite signs. The spherical charged region cannot move. *For this question, it may be useful to know that the electrical field inside a uniformly charged shell is 0.*

   Mass of electron: 9.1×10^{-31} *kg*

   Mass of proton: 1.67×10^{-27} *kg*

8. Two small balls of mass m are hanging from a string of length l. Initially, each ball has a positive charge q_0. The distance between the balls is then $r << l$. The charge slowly leaks out to the surroundings so that the charge changes with time: $q(t) = q_0(1 - bt)^{3/2}$, where b is a constant. Therefore, the balls will approach each other. Find the speed with which they approach each other. You can assume that the charge leaks out slowly, so that you can ignore the acceleration and assume that they are in equilibrium at all times. Hint: You can use the fact that when $r << l$, $\tan\theta \approx \sin\theta = \frac{r}{2l}$.

9. Four point charges $+q$ are at rest on an insulating surface, each in a corner of a square with side s. A fifth point charge $+Q$ is located at a height h above the center of the square. Determine h so that the sum of the forces from the four stationary charges is as large as possible.

10. A small bead of mass m and charge q can slide without friction on a rigid, insulating circular hoop of radius R. The hoop lies in a vertical plane. Gravity acts downward with acceleration g. A uniform, static electric field $\mathbf{E} = E\,\hat{\mathbf{x}}$ points horizontally to the right. Let θ be the angle of the bead measured from the *downward* vertical (so $\theta = 0$ at the bottom of the hoop).

    (a) Find an expression for the angular acceleration $\ddot{\theta}$ in terms of m, q, R, g, E, and θ.

    (b) Find all equilibrium angle(s) θ_*.

Answers

1. (a) $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{(1\,\mathrm{C})(1\,\mathrm{C})}{(1\,\mathrm{m})^2} \approx 8.99\times10^9$ N.

   (b) $m = \sqrt{\dfrac{F}{G}} \approx 1.1604\times10^{10}$ kg (each mass).

2. (a) $\vec{F} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{r^2}\,\hat{\mathbf{r}}$.

   (b) $\mathrm{d}W = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{r^2}\,\mathrm{d}r$.

   (c) $W = -\displaystyle\int_r^\infty \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{r^2}\,\mathrm{d}r$.

   (d) $W = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{r}$.

   (e) $V = \dfrac{W}{q_1} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_2}{r}$.

3. $q \approx 5.20\times10^{-8}$ C, $\quad 2q \approx 1.04\times10^{-7}$ C.

4. $r = \sqrt{\dfrac{(M+m)\,k\,Qq}{E\,(mQ+Mq)}}, \quad k = \dfrac{1}{4\pi\varepsilon_0}$.

5. $\dfrac{K_{e^+}}{K_p} \approx 1 + 4\sqrt{2} \;\approx\; 6.66$.

6. proton energy fraction $= \dfrac{m_e}{m_p+m_e} = \dfrac{1}{1836+1} \approx 5.45\times10^{-4}$ (= 0.0545%).

7. $r_{\max} = \left(\dfrac{1}{R} + \dfrac{K_0}{k\,qQ}\right)^{-1}$ for $K_0 < \dfrac{k|qQ|}{R}$; $\quad r_{\max} = \infty$ otherwise, $\quad k = \dfrac{1}{4\pi\varepsilon_0}$.

8. $v = \left|\dfrac{\mathrm{d}r}{\mathrm{d}t}\right| = b\left(\dfrac{2l}{4\pi\varepsilon_0\,m\,g}\right)^{1/3} q_0^{2/3}$.

9. $h = \dfrac{s}{2}$.

10. (a) $\ddot{\theta} = -\dfrac{g}{R}\sin\theta + \dfrac{qE}{mR}\cos\theta$.

    (b) $\theta^* = \arctan\!\left(\dfrac{qE}{mg}\right) + n\pi, \quad n \in Z$.